# Algebra Intensification Project

The Urban Mathematics Leadership Network

Strengthening Mathematics Learning for All Students

A collaboration between
The Charles A. Dana Center at The University of Texas at Austin
Achieve, Inc., and The Aspen Institute

## Goals for Session

- Part I: Provide brief introduction about project background, context, and key design features
- Part II: Illustrate overall lesson structure and provide opportunity to examine some specific lesson materials
- Part III: Illustrate a classroom implementation
- Part IV: Provide opportunities for questions and discussion

## Part I: Program Overview

- Project context and background
- Project timeline
- Key design features
- Course scope and sequence

## Program Context: Need & Overarching Design Principles

- Need identified by UMLN district leaders; discussed at UMLN annual meeting
- Designing for two populations
  - Under-prepared algebra students
  - Teachers of under-prepared algebra students
- Designed for use in double-period high school Algebra 1 courses
- Designing for use at scale in urban districts and elsewhere

## Program Context: Project Partners/Support

- Charles A. Dana Center, University of Texas at Austin
- University of Illinois at Chicago, Learning Sciences Research Institute
- Agile Mind, Inc.

- Project Funding
  - Grants that support UMLN from Dana, Gates, and Carnegie Foundations
  - Chicago Community Trust

## Project Timeline

- 2007-2008:
  - Planning grant from Chicago Community Trust
    - Establish Design Team to guide project
    - Study issues and existing interventions
    - Flesh out possible project design; develop prototype materials
  - Proposed project design reviewed by UMLN leaders
- 2008-2009:
  - Prototype units developed
  - Prototype units (3) piloted in four high schools in UMLN districts: Chicago, Austin, (Evanston)
  - Funding sought for full program implementation

## Project Timeline

- 2009-2010:
  - Revise 3 prototype units based on teacher feedback, classroom observations
  - Complete development of 2-4 additional prototype units
  - Pilot prototype units in an expanded set of UMLN districts schools
  - Obtain funding for full program development

## Project Design: An Architecture for Intensification

- Under-prepared students need more time.
- More time alone is not enough.
  - Rigorous algebra core
  - Pedagogy designed for struggling and special-needs learners
  - Effective strategies to address misconceptions
  - Social/motivation issues
  - Literacy support
  - Strong assessment program
  - Structures for teacher support

## Program Design: Key Design Features

- Algebra core
- Efficient review/repair strategies
- Ongoing, distributed practice
- Incorporation of social-psychological learning interventions
- Supports for enactments of high cognitive demand tasks
- Tools that help students organize information and support metacognitive awareness
- Enhanced formative assessments strategies
- Explicit supports for literacy and language development

## Program Design: Course Scope and Sequence

**(scope and sequence chart available in on-line resources)**

- **Unit 0**: Getting started
- **Unit 1**: Developing mathematical models
- **Unit 2**: Recognizing linear and nonlinear patterns
- **Unit 3**: Rate of change
- **Unit 4**: Linear functions
- **Unit 5**: Linear equations and inequalities
- **Unit 6**: Systems of linear equations
- **Unit 7**: Nonlinear functions: exponential and quadratic functions
- **Unit 8**: Quadratic equations
- **Unit 9**: Other nonlinear functions

**Intensified Algebra I - Working Scope and Sequence/Agile Mind, Inc.TM/Charles A. Dana Center, January, 2009**

| Intensified Algebra Topics | Time Allotted | | Topic Description | Mathematics Goals and Objectives | Youth Development Goals and Objectives |
|---|---|---|---|---|---|
| | Weeks | Days | | | |
| **Unit 0: Getting Started** | | | | | |
| 1: Exploring problem-solving strategies | 0.8 | 4 | Students begin to experience collaboration as a strategy to solve problems. They share problem-solving strategies as they explore problems that have single and multiple solutions. | 1. Explore and apply different strategies to solve problems in real-life contexts.<br>2. Understand that for any given problem there may be multiple pathways to the same solution, that there may be multiple solutions, and that multiple pathways and solutions are equally valid. | 1. Begin to get to know on another and build trust.<br>2. Begin to learn about the goals and objectives of the program.<br>3. Develop and share expectations for partner (and group) work.<br>4. Recognize the value of sharing solution strategies and working together to solve problems in real-life contexts. |
| 2: Getting smarter through problem solving | 0.6 | 3 | This topic introduces students to the ideas of malleable intelligence and brain growth through learning. Students continue to develop problem-solving strategies as they extend their understanding of patterns by exploring polygon trains. | 1. Understand that mathematical problem solving means different things to different people and that mathematical problem solving means more than just solving word problems or "doing something with the numbers."<br>2. Understand that for any given problem there may be multiple pathways to the same solution, that there may be multiple solutions, and that multiple pathways and solutions are equally valid.<br>3. Recognize, describe and extend patterns.<br>4. Use symbols to represent unknowns and variables.<br>5. Represent numerical patterns using input/output tables. | 1. Understand the difference between an entity (fixed) and incremental (malleable) theory of intelligence.<br>2. Learn basic information about neurons, dendrites, axons, and synapses. |
| 3: Problem solving, effort, and your brain | 1.2 | 6 | Students learn about the concept of working harder to get smarter, and they apply this idea to learning mathematics. They also learn that effective communication is an interaction between the giver and getter of information. They then apply this understanding as they solve problems in this topic. They continue to explore multiple solution strategies and learn how being able to approach problems in multiple ways can help maintain motivation in problem solving. | 1. Understand that for any given problem there may be multiple pathways to the same solution, that there may be multiple solutions, and that multiple pathways and solutions are equally valid. | 1. Understand the benefits of effective effort and the role of effort in creating new connections in the brain and increasing "smartness".<br>2. Recognize the value of sharing solution strategies and working together to solve problems in real-life contexts. |

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| **Unit 1: Developing mathematical models** | | | | | |
| 4: Representing relationships in multiple ways | 1.4 | 7 | This topic develops a key theme of the course: Relationships between variables can be represented using words, tables, graphs, or symbols. Students are formally introduced to different ways to represent patterns and relationships and begin to connect various representations of proportional and non-proportional situations to one another (verbal, numeric, graphical, algebraic). They also extend their understanding of multiple representations in a way that will pay big dividends in Algebra I: They begin to learn to generate other, related representations when given a single representation of a pattern or relationship. | 1. Describe relationships among quantities.<br>2. Represent relationships among quantities using concrete models, tables, graphs, diagrams, and equations.<br>3. Use symbols to represent unknowns and variables.<br>4. Represent generalizations from patterns algebraically.<br>5. Use algebraic properties to simplify expressions. | |
| 5. Thinking about algebra | 0.6 | 3 | Students further develop their problem-solving capabilities and their algebraic thinking by working on a non-routine problem, Consecutive Sums. Students also explicitly explore the use of metacognitive strategies to improve their problem solving and learning. | | 1. Understand that not all effort is equal - there are things we can do that result in more effective effort.<br>2. Be able to use the metacognitive strategies plan, monitor, evaluate, and loop back to help make effort effective. |
| 6: Variables and functions | 1.4 | 7 | Students are informally introduced to the concept of function as a dependency relationship between two variables, in which one depends on the other in a systematic way. Students extend their growing understanding of multiple representations and use them to represent functions involving proportional and non-proportional linear relationships algebraically, numerically, graphically and verbally. | 1. Demonstrate understanding of the concept of a function as a systematic relationship between two variables.<br>2. Begin to identify independent and dependent variables in functional relationships.<br>3. Represent functions using words, tables, symbols, and graphs.<br>4. Gather, organize, and interpret data for representations of functional relationships. | |

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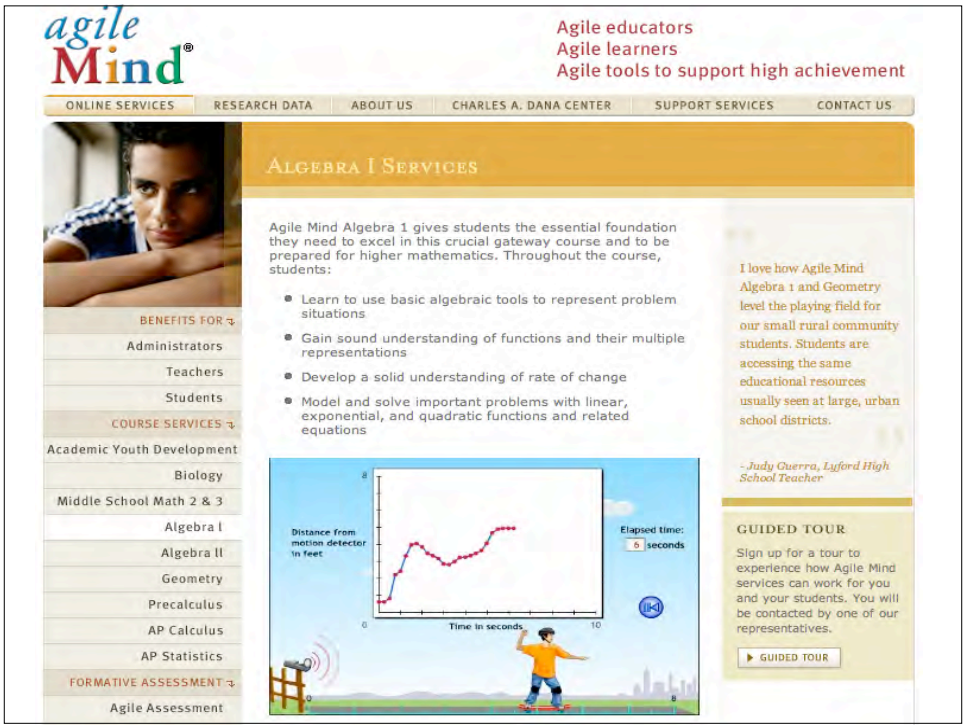
| Unit 6: Systems of linear equations | | | | | |
| 19: Formulating and solving systems | 1.2 | 6 | Systems of linear equations, in which two conditions apply to a situation, and thus must be modeled with two equations, are introduced in this topic. Students learn to set up a system, solve it using graphs and tables, and check the solution for reasonableness. | 1. Starting with a written description (word problem), students will identify the 2 variables needed to solve the problem and write a system of linear equations in those 2 variables to model the situation.<br>2. Students will solve a system by making an appropriate table of values by hand and/or on a calculator.<br>3. Students will solve a system by graphing the equations and finding their point of intersection.<br>4. Students will use a systematic process to solve word problems.<br>5. Students will check their results by substituting into *both* equations of the system. | |
| 20: Mindset | 0.6 | 3 | Students continue to develop their problem-solving capabilities and algebraic thinking by working on a non-routine problem, The Icicles Problem. They also explore the idea of "mindset" and how it can affect their success | | 1. Learn about the idea of "mindset."<br>2. Understand how different mindsets can lead to behaviors that either support or hinder one's school performance. |
| 21: Other methods for solving systems | 1.2 | 6 | Continuing with the exploration of systems of two linear equations, this topic introduces two algebraic methods for solving systems: the substitution method and the linear combination method. Students begin to see when to use each method, and how to interpret the results each method yields. | 1. Solve systems of linear equations using the substitution method.<br>2. Solve systems of linear equations using the linear combination method.<br>3. Understand the logic behind the linear combination method.<br>4. Recognize dependent and inconsistent systems and write the solution set of each. | |

# Part II: Lesson Structure

- Specific Components
- Role of Agile Mind On-line Resources
- Examine Teacher & Student Materials -- 1 Day
- Reactions & Discussion of Lesson Structure

## Lesson Structure Components

- Daily Preview
- Opener
- Core Learning Activities
  - Agile Mind online service
- Process Homework
- Consolidation Activities
- Staying Sharp
- Homework



1.2 Introducing Systems of Equations

Ms. Salinas, who is in charge of sales for Opportunity Company, knows that a certain job will take 8 hours to complete. She has budgeted $80.00 for the job.

The supervisor, who is paid $15 per hour, will start the job, in order to plan and organize it. Then, her assistant, who is paid $7 per hour, will take over and complete the job.

Ms. Salinas needs to figure out how long each person should work in order for the company's costs to meet her budget and time estimates.

Work with your partner to determine how long each person should work to meet both of Ms. Salinas' conditions.

Students will likely employ a variety of informal methods to reach a solution to the problem: **Ms. Salinas should work for 3 hours and her assistant shold work for 5 hours.**

A method for more formally approaching the problem with the use of algebra will be developed as the activity continues.

# Examine A Daily Lesson

- Got to the on-line resources for Intensified Algebra
- Open the resource called Lesson Example
- Spend 20 minutes reviewing the full set of materials
  - Notice how the structure follows the design components
- Make notes about specifics -- be prepared to discuss observations and reactions

LESSON RESOURCES:

- Computer with projection device and Internet connection
- Graphing calculators
- Large chart paper
- Masking tape
- Chart markers
- Whiteboards, dry erase markers, and erasers

**Overview of the day and time management**

| Time | Activity | Goal | Student Pages | AM Screens |
|---|---|---|---|---|
| 5 min | 1: Opener: The Bike and Skateboard Problem Solution | • To use a familiar problem context as an entry to the concept of systems of equations | p. 5 | NA |
| 25 min | 2: Introducing Systems of Equations | • To introduce the concept of linear systems of equations | pp. 6-8 | Topic 7 *Overview*, screens 1-5 |
| 10 min | 3: Process Homework | • To learn from reviewing the homework due today | p. 9 | NA |
| 20 min | 4: Building a System of Equations Problem | • To understand a system of equations problem as a set of interacting conditions | pp. 10-11 | NA |
| 15 min | 5: Square Box Problems as Systems of Equations | • To connect a familiar tool and associated thinking process to systems of equations | pp. 12-14 | NA |
| 5 min | 6: Introduce Homework | • To understand tonight's homework assignment | SS: p. 15<br>HW pp.17-18 | NA |

# Part III: Program in Action

# Part IV: Questions, Comments, Issues of Application to MN Context

# Components of a Lesson

- ***Daily Preview:*** Outline of day's activities that explicitly describes what students will be doing and the purpose of each activity. The preview helps students organize their thinking for the lesson and see the connections among lesson components.

- ***Opener:*** *A* 5-10 minute daily warm-up "routine" for transitioning into class work. The goal is to help focus students on the upcoming lesson and access relevant prior knowledge, and/or to provide teachers with formative assessment data. The warm ups typically involve math problems around concepts or skills needed for the lesson but may also be questions for private reflection and/or partner discussion.

- ***Core Learning Activities:*** 20-30 minutes of instruction adapted from AM lessons to promote learning of essential algebra content. Activities typically feature high-cognitive-demand tasks situated in real-life contexts along with online animations of mathematically important aspects of the tasks.
- Tasks are augmented by regular use of routines that provide new ways for students to organize and access the content, e.g., graphic organizers to help students make connections among concepts; triple-entry journals to actively access and reflect upon prior knowledge and/or process new mathematics vocabulary; a"think-pair-share" routine for partner work; and explicit reading instructional strategies to aid comprehension of problems and help students monitor understanding as they read the algebra lessons.
- Routines to support frequent formative assessments (e.g., individual whiteboard work) to help teachers and students monitor learning are also included.

- ***Process Homework:*** 5-10 minute partner routine to review previous night's homework. It promotes communication among students about mathematics and their mathematical thinking, and also teaches them to take ownership of their own learning. Students process their mid-unit and end-of unit assessments with their partners using an expanded version of this routine.

- ***Consolidation Activities:*** 20-30 minutes of instruction designed to review/repair prior knowledge required for upcoming lessons (i.e., preview the prealgebra content and correct misconceptions), provide additional opportunities for practice, and/or address youth development topics. These are typically partner activities specifically designed to make students confront common misconceptions.

- ***Staying Sharps:*** A daily set of six short problems that provide distributed practice with algebra and pre-algebra skills and use of metacognitive tools. Staying Sharps are also used to preview up-coming content; i.e., they help students review relevant prerequisite knowledge and also provide formative assessment data to teachers about their students' knowledge of perquisite concepts and skills. Teachers typically assign Staying Sharps either as homework or to be done during the consolidation period.

- ***Homework:*** Roughly 30 minutes of additional work outside of class time to help develop students' confidence and abilities to work independently in mathematics and to provide additional practice.