Intensified Algebra I:
A UMLN Algebra Intensification Program

The Algebra Intensification Program is supported by grants from the Gates Foundation, the Carnegie Foundation, the Dana Foundation, and the Chicago Community Trust.
Course Structure and Content

Course content is organized into ten instructional units, each consisting of roughly three weeks of instruction. Units are subdivided into three “topics”—each of which includes a sequence of lessons about specific mathematics content. Typically, the first and third topics in a unit provide 5-6 days each of instruction on specific themes of mathematics, while the middle topic addresses youth-development ideas and builds students’ algebraic thinking capabilities via engagement in solving non-routine problems.

Lesson Structure and Content

Each daily lesson is written for 80-minute blocks, which is the typical length of double-period classes in UMLN districts and elsewhere.

A typical lesson has the following components:

- **Daily Preview**: Outline of day’s activities that explicitly describes what students will be doing and the purpose of each activity. The preview helps students organize their thinking for the lesson and see the connections among lesson components.

- **Opener**: A 5-10 minute daily warm-up “routine” for transitioning into class work. The goal is to help focus students on the upcoming lesson and access relevant prior knowledge, and to provide teachers with formative assessment data. The warm-ups typically involve mathematics problems involving concepts or skills needed for the lesson but may also involve questions for private reflection and/or partner discussion.

- **Core Learning Activities**: 20-30 minutes of instruction adapted from Agile Mind lessons to promote learning of essential algebra content. Activities typically feature high-cognitive-demand tasks situated in real-life contexts along with online animations of mathematically important aspects of the tasks. Tasks are augmented by regular use of routines that provide new ways for students to organize and access the content, e.g., graphic organizers to help students make connections among concepts; triple-entry journals to actively access and reflect upon prior knowledge and/or process new mathematics vocabulary; a “think-pair-share” routine for partner work; and explicit reading instructional strategies to aid comprehension of problems and help students monitor understanding as they read the algebra lessons. Routines to support frequent formative assessments (e.g., individual whiteboard work) to help teachers and students monitor learning are also included.

- **Process Homework**: 5-10 minute partner routine to review the previous lesson’s homework. It promotes communication among students about mathematics and their mathematical thinking, and also teaches students to take ownership of their own learning. Students process their mid-unit and end-of unit assessments with their partners using an expanded version of this routine.
• **Consolidation Activities:** 20-30 minutes of instruction designed to review/repair prior knowledge required for upcoming lessons (i.e., preview the algebra content and correct misconceptions), provide additional opportunities for practice and to deepen conceptual understanding, and/or address youth development topics. These are typically designed as partner activities.

• **Staying Sharps:** A daily set of six short problems that provides distributed practice with algebra and prealgebra skills and use of metacognitive tools. Staying Sharps are also used to preview upcoming content; i.e., they help students review relevant prerequisite knowledge and also provide formative assessment data to teachers about their students’ knowledge of prerequisite concepts and skills. Teachers typically assign Staying Sharps either as homework or to be done during the consolidation period.

• **Homework:** Roughly 30 minutes of additional work outside of class time to help develop students’ confidence and abilities to work independently in mathematics and to provide additional practice.

**Lesson Design Process**

In adapting Agile Mind algebra activities for use with underprepared students, we consider: How can we foster student engagement? How can we further promote development of conceptual understanding? How can we prompt productive, focused student interactions to support knowledge construction? How can we better support acquisition of mathematical language? How can we make student thinking more visible? This analysis informs changes to the algebra lessons themselves, including the sequencing of the content, and determines the content of the supporting student pages and Advice for Instruction to the teacher.

We then consider a series of questions to determine the content of the other lesson components that will best support the core lesson, e.g., What prior knowledge will facilitate students’ learning of the core lesson? What prerequisite skills are needed? What common misconceptions for this mathematics idea need to be addressed? What cognitive and sociocultural considerations might impact how struggling learners perceive the mathematics? How can the learning environment be shaped to emulate the practices of good problem solvers and enhance students’ identities as capable and confident mathematics learners?

The answers to these questions determine content for the openers, consolidation activities, Staying Sharps, and homework, as well as inform the support structures and learning environment design. As each lesson is developed and sequenced, we are constantly looking for ways to build in appropriate supports in a timely manner to ensure that students will have opportunities to successfully access the mathematics content and develop the disciplinary practices of representation, communication, and problem solving.

As part of the design process, we develop a set of printed student pages that will be produced as consumable booklets. The student pages support the thinking/learning in the lesson in various ways, e.g., provide opportunities for students to record answers to scaffolding prompts, partner discussion questions, solutions to problems, or double-column journal entries to facilitate access to important vocabulary or understandings about themselves as learners, as
well as to provide a reference for learning that students can consult for homework or in making connections to new lessons. We also determine what questions, actions, and supports should be provided in the Advice for Instruction, keeping in mind that many teachers of the double-period course are relatively inexperienced teachers.

Sample Lesson

For illustrative purposes, a sample lesson is included in the subsequent pages. The Advice for Instruction for Day 1 of “Unit 6: Systems of Equations” are provided. [Note: The Advice for Instruction contains reduced student pages (with answers).]
Unit 6: Systems of Linear Equations

This unit introduces the important mathematical idea of solving systems of linear equations. You will connect your work with other parts of the course, namely multiple representations and solving linear equations. You will learn how to model situations involving linear systems and use a variety of methods for solving linear systems, including

- tables
- graphs
- substitution, and
- linear combination.

You will learn about the idea of mindset and how it can affect your learning and academic performance.

Finally, you will continue to use the routines and structures of the course, including partner work and Staying Sharps, to continue to grow and develop as a mathematics learner.

Unit Outline

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<th>Topic</th>
<th>Days</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>7. Formulating and Solving Systems</td>
<td>Days 1-7</td>
<td>Systems of linear equations, in which two conditions apply to a situation, and thus must be modeled with two equations, are introduced in this topic. You will learn to set up a system, solve it using graphs and tables, and check the solution for reasonableness.</td>
</tr>
<tr>
<td>8. Mindset</td>
<td>Days 7-9</td>
<td>You will continue to develop your problem-solving capabilities and your algebraic thinking by working on a non-routine problem, The Icos Problem. You will also explore the idea of “mindset” and how it can affect your success as a student.</td>
</tr>
<tr>
<td>9. Other Methods for Solving Systems</td>
<td>Days 10-15</td>
<td>Continuing with the exploration of systems of two linear equations, in this topic you will be introduced to two algebraic methods for solving systems: the substitution method and the linear combination method. You will begin to see when to use each method, and how to interpret the results each method yields.</td>
</tr>
</tbody>
</table>
Page 1 in the student booklet provides an overview of Unit 6. The unit consists of three topics (Topics 7, 8, and 9). A description of each topic is provided in the table for students.

Unit 6 Overview for Teachers:

Topic 7: This topic introduces the concept of a system of equations and makes connections to other learning, including familiar problem contexts and previous topics in the course, like multiple representations of function. The first part of this topic concentrates on learning to model situations with systems of equations; deriving a process for solving systems problems; and the use of tables and graphs as a means of solving linear systems of equations.

Topic 8: This topic introduces the youth development topic of mindsets and connects this idea to a problem-solving experience that makes use of an extended, non-routine problem called the Icicles problem. Students revisit the problem-solving routine. Goals include fostering algebraic thinking, mathematical problem solving, and the “learning to learn” topic of how one’s mindset can affect learning.

Topic 9: This topic focuses on developing students’ understanding and proficiency with solving linear systems of equations using two methods: the substitution method and the linear combination method. For the Intensified Algebra course, a decision was made to give slightly more treatment to the substitution method over the linear combination method, with the idea that the nuances of the linear combination can be more fully addressed in Algebra II. The learning expectation for this topic is that students become proficient—in addition to solving systems via substitution—with solving basic systems using the linear combination method (e.g., problems where one equation is multiplied by a factor; this constitutes the “upper limit” of problems for which students are expected to master for Topic 9.)

Note: This unit also includes an optional topic, Topic 9a, that extends the development of ideas related to solving linear systems of equations using the linear combination method. This topic might be appropriate for schools/districts for which state standards for Algebra I call for a deeper understanding of the linear combination method. The topic addresses more challenging systems problems as well as nuances related to this solution method (e.g., the idea that in certain situations, it is more efficient to apply the linear combination method twice—once by manipulating the x-coefficients and then by manipulating the y-coefficients—in contrast to the method developed in Topic 9, of using substitution to solve for the second variable after linear combination is used to solve for the first variable).
Topic 7: Formulating and Solving Systems

This topic introduces systems of linear equations to solve problems that involve two conditions and so must be modeled with two equations. You will learn to set up a system, solve it using graphs and tables, and check the solution for reasonableness.
Page 3 in the student booklet provides an overview of Topic 7: Formulating and Solving Systems.

Topic 7 Overview for Teachers:

Day 1: This lesson introduces the concept of a system of equations. Students explore a new problem—the Supervisor problem—as well as revisit familiar problems—the Bike and Skateboard problem and Square Box problems—that are modeled with systems of equations. Goals are for students to understand that (1) a system of equations describes a set of conditions to be met, and (2) a solution(s) to a system consists of values that satisfy these conditions.

Day 2: The focus of the lesson is modeling situations with systems of equations and using informal means to find solutions. A six-step process for solving systems is introduced; the goal of this lesson is for students to develop proficiency in the first part of this process, steps 1-3, which involves defining variables to represent the unknowns and setting up equations to represent conditions/constraints.

Day 3: The lesson focuses on using tables to solve systems of equations and on using steps 4-6 in the six-step process. In Part 1, students solve systems for which tables is an effective method; i.e., systems with solutions that consist of integer values easily found by using a table. In Part 2, students experience the limitations of using tables to solve systems. They are presented with systems for which the use of tables is not as straightforward; i.e., ones for which they need to interpolate between table entries to find a solution. The goals of the lesson are for students to (1) learn to use tables as a solution method, and (2) recognize the limitations of this method.

Day 4: This lesson focuses on using graphs to solve systems of equations. Students learn that the solution to a system is represented graphically by the point of intersection of the lines depicting the constraints of the problem. Students compare using tables and graphs as solution methods using two familiar problems, the Swamp Problem from Day 2 and the Roses Problem from Unit 1. They also explore limitations of graphing by hand to solve systems. The goals of the lesson are for students to (1) learn to use graphs as a solution method, i.e., that the point of intersection of the lines on the graph represent the solution to the system, and (2) to recognize the limitations of graphing by hand as a solution method.

Day 5: Students continue to explore graphing as a solution method, learning to graph the system with technology and learning how to use the calculator’s built-in capabilities to solve systems problems. They also connect graphing systems to coordinate geometry: they explore when systems produce parallel lines and intersecting lines, and what that means in terms of the solution of the system. The goals of the lesson are for students to (1) use their graphing calculator’s capabilities to solve systems problems, (2) distinguish between systems that produce intersecting and parallel lines, and (3) recognize that intersecting lines mean a system has a single, unique solution and that parallel lines mean that the system has no solution.

Day 6: Students create their own systems problem, then solve it using either tables or graphs. They also take the Agile Mind on-line assessments to monitor their learning in this unit before the mid-unit assessment on Day 7.

Day 7: Students take the mid-unit assessment in the first part of the lesson. In the second part of the lesson, the concept of mindset, the focus of Topic 8, is introduced.
DAY 1 LESSON PLAN: Introducing Systems of Linear Equations

LESSON RESOURCES:

- Computer with projection device and Internet connection
- Graphing calculators
- Large chart paper
- Masking tape
- Chart markers
- Whiteboards, dry erase markers, and erasers

Overview of the day and time management

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<tr>
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<th>Goal</th>
<th>Student Pages</th>
<th>AM Screens</th>
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<td>5 min</td>
<td>1: Opener: The Bike and Skateboard Problem Solution</td>
<td>To use a familiar problem context as an entry to the concept of systems of equations</td>
<td>p. 5</td>
<td>NA</td>
</tr>
<tr>
<td>25 min</td>
<td>2: Introducing Systems of Equations</td>
<td>To introduce the concept of linear systems of equations</td>
<td>pp. 6-8</td>
<td>Topic 7 Overview, screens 1-5</td>
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<tr>
<td>10 min</td>
<td>3: Process Homework</td>
<td>To learn from reviewing the homework due today</td>
<td>p. 9</td>
<td>NA</td>
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<tr>
<td>20 min</td>
<td>4: Building a System of Equations Problem</td>
<td>To understand a system of equations problem as a set of interacting conditions</td>
<td>pp. 10-11</td>
<td>NA</td>
</tr>
<tr>
<td>15 min</td>
<td>5: Square Box Problems as Systems of Equations</td>
<td>To connect a familiar tool and associated thinking process to systems of equations</td>
<td>pp. 12-14</td>
<td>NA</td>
</tr>
<tr>
<td>5 min</td>
<td>6: Introduce Homework</td>
<td>To understand tonight’s homework assignment</td>
<td>SS: p. 15 HW pp.17-18</td>
<td>NA</td>
</tr>
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Topic 7: Formulating and Solving Systems

1.0 Preview of the Day

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<td>To use a familiar problem context as an entry to the concept of systems of equations</td>
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<tr>
<td>Solution</td>
<td></td>
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<tr>
<td>2: Introducing Systems of Equations</td>
<td>To introduce the concept of linear systems of equations</td>
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<td>3: Process Homework</td>
<td>To learn from reviewing the homework due today</td>
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1.1 OPENER: The Bike and Skateboard Problem Solution

Do you recall the “Bike and Skateboard” Problem from earlier in the course? The problem is summarized below along with the solution to the problem.

Uncle Eddie asked McKeena and Lina to order 54 new wheels for the 21 skateboards and bicycles in his repair shop. How many bicycles and how many skateboards are in Uncle Eddie’s shop?

There are 15 bicycles in the shop.

There are 6 skateboards in the shop.

Task: Show or explain why the combination of 15 bikes and 6 skateboards is the correct solution to this problem.

Sample answer: The combination satisfies both conditions in the problem:

-- There are 21 total objects (15 bikes + 6 skateboards).
-- There are 54 total wheels [(15 bikes · 2 wheels) + (6 skateboards · 4 wheels)].
ACTIVITY 0: Preview of the Day

1. Preview the day’s lesson for students so that they are aware of the activities in the lesson as well as the learning purpose(s) for each activity.

2. You may wish to project page 5 of the student pages, which contains the Preview of the Day. Some teachers find it helpful to preview the lesson before assigning the opener; other teachers find it more useful to preview the lesson after students have worked on the opener.

ACTIVITY 1: Opener—The Bike and Skateboard Problem Solution (5 minutes; p. 5)

1. The Bike and Skateboard problem from earlier in the course is used as a familiar context to informally introduce the notion of a system of equations.

2. Students are asked to explain in writing why the combination of 15 bicycles and 6 skateboards is the correct solution to the problem.

3. Allow students several minutes to work on the opener, then process it as a class. A key idea to emphasize in the discussion is the fact that this combination satisfies both conditions of the problem. As part of the processing, be sure that students explicitly show that both conditions are satisfied, showing that this combination results in 54 wheels and 21 objects.

4. Tell students that when they solved this problem earlier in the course, they actually solved a “system of equations,” which is the topic of study for this unit. Do not spend time formally connecting the skateboard problem to the concept of a system of equations. This will be done in upcoming activities. In fact, an activity on Day 2 presents students with yet another version of the bike and skateboard problem and asks students to approach it from a system of equations perspective.

5. While the purpose of the opener is to provide a point of entry to systems of equations and to build students’ confidence in working with this new topic, you may want to emphasize that students will be studying this topic in depth and building multiple algebraic strategies for solving systems of equations problems (i.e., that students will learn multiple, “sophisticated” strategies and methods for solving these types of problems and that learning and having access to these different strategies will become important as the problems increase in difficulty).
Students will likely employ a variety of informal methods to reach a solution to the problem: **Ms. Salinas should work for 3 hours and her assistant should work for 5 hours.**

A method for more formally approaching the problem with the use of algebra will be developed as the activity continues.
ACTIVITY 2: Introducing Systems of Equations (25 minutes; pp 6-8; AM Topic 7 Overview)

Overview:

- Students are introduced to systems of equations; students first work on a system-like problem, using whatever approach they want to solve the problem (similar to their work on the bike and skateboard problem). Students think about the process that they used to solve the system and then are asked to write a pair of equations that model the conditions in the problem and to treat this pair of equations as a system. Students are asked to attempt to solve the system. In this way, a formalized approach to systems of equations is “overlaid” on the problem, which will form the foundation for their developing understanding of ways to approach systems of linear equations. Students are asked to work with their partners on various stages of the problem; the Agile Mind screens are used on several occasions in this activity to check students’ work (i.e., the activity attempts to place the “cognitive load” on students).

1. Show screen 1 of the Overview and give students about 10 minutes to try to solve the problem. Space is provided on student p. 6 for students to work on the problem. Some questions that may help as you circulate while students are working on the problem:
   - How many things are you looking for?
   - Could you write equations if you are allowed to use 2 variables instead of one?

2. Bring the class together to discuss solutions and approaches to the problem. Tell students that the problem will serve as an introduction to systems of equations. Acknowledge that while there are several valid ways to solve the problem (including guess-and-check), students will be learning a method involving “systems of linear equations.” You can tell students that when problems like this become increasingly more difficult, it is useful to have a toolbox of methods and strategies for approaching these types of problems. This is the motivation for studying several different methods in this unit. Finally, mention to students that they will be making connections in this unit to other units, including multiple representations of functions and solving linear equations.

3. Show screen 2. Inform students that they will now attempt to write and solve a system of equations for this problem and that the animation will allow them to check their work.
Ms. Salinas realizes she can express these two conditions with two equations. She lets \( s \) represent the number of hours the supervisor works and \( a \) represent the number of hours the assistant works.

1. Write an equation to show the sum for the total hours worked.

\[
s + a = 8
\]

2. Write an equation for the total cost of the job using the hourly pay for each person.

\[
15s + 7a = 80
\]

**LANGUAGE NOTE**

The word “system” can have several meanings. Sometimes people use the word “system” when talking about a set of rules or procedures for accomplishing a goal: “John has developed a good system for washing the dishes. First he scrubs off the food, then he rinses the plates in the sink, and then he loads the dishes in the dishwasher.”

People also use the word “system” to talk about related elements that make up a whole: “Dorothy bought a new stereo system that includes a tuner, speakers, and a CD player.”

In this topic, the word “system” is used to describe a set of two or more equations with two or more variables for which to solve. For example, look at the system of equations from this Overview:

Enter your equations from above in the box to the left.
4. Before playing the animation, allow student an opportunity to work with their partners to write equations to try to model the two conditions in the problem. Space is provided at the top of p. 7 in the student booklet.

5. Then play the animation as a means of checking students’ answers and solidifying their understanding. Add explanation as you see fit as you step through the animation. Check for student understanding as you proceed through the animation by asking questions of students.
6. **Show screen 3.** This screen formally introduces the vocabulary *system of equations.*

![Screen 3](image)

6. **System of Equations**

This is a system of equations. A *system of equations* is a set of two or more equations with two or more variables for which to solve. In algebra, you can form systems by writing equations to model different aspects of the same situation.

7. **Language note:** Students may need further clarification and practice with core vocabulary presented on screen 3. This language note appears on page 7 in the student booklet. There is also a “Language note” icon on screen 3. Discuss this language note as a class. You can begin by asking students to explain their understanding of the term “system” as it is used in everyday language. Ask students for their own examples to illustrate the different meanings that they generate. Then make a distinction between the everyday uses of the term and the way in which “system” is used in mathematics. Encourage students to take additional notes next to the language note that appears on student p. 7 in their books.

![Screen 3 Language Note](image)

### Screen 3 Language Note

The word “system” can have several meanings. Sometimes people use the word “system” when talking about a set of rules or procedures for accomplishing a goal: “John has developed a good system for washing the dishes. First he scraps off the food, then he rinses the plates in the sink, and then he loads the dishes in the dishwasher.”

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In this topic, the word “system” is used to describe a set of two or more equations with two or more variables for which to solve. For example, look at the system of equations from this OverView:

\[
\begin{align*}
  s + a &= 8 \\
  15s + 7a &= 80
\end{align*}
\]

In this system of equations, the two variables are \( s \) and \( a \), where

\( s \) = the number of hours that Ms. Salinas will work, and

\( a \) = the number of hours that her assistant will work.

Put all this together, and you have a system of equations in two variables.
3. Together, these equations form a __________ of __________ in two variables.

4. Solve the Supervisor Problem with your partner, then express your solution in two ways:
   a. In words: 
      **Sample answer:** The supervisor should work for 3 hours and the assistant should work for 5 hours.

   b. As an ordered pair [with the number of hours the supervisor works as the first ordinate and the number of hours that the assistant works as the second ordinate—that is, in the form \((s, a)\)].
      \( (3, 5) \)

5. In general, what is a solution to a system of equations in two variables?
   [Hint: In addition to thinking about the specific solution to The Supervisor Problem, you might find it helpful to also think about the Opener—that is, why 15 bikes and 6 skateboards is the solution to that problem.]
   **Sample answer:** A solution to a system of equations is an ordered pair that will make both equations true.

6. Check your solution to The Supervisor Problem by substituting the ordered pair that you and your partner got back into the original equations and then by evaluating each equation using this ordered pair.

   \[
   \begin{align*}
   s + a &= 8 \\
   3 + 5 &= 8 & \checkmark \\
   15s + 7a &= 80 \\
   15(3) + 7(5) &= 80 & \checkmark \\
   45 + 35 &= 80 & \checkmark 
   \end{align*}
   \]
8. Once students have written the system of equations, allow them time to try to solve the system. Students should now work with their partner to complete the questions on student page 8.

9. Use screen 4, the animation on screen 4, and screen 5 to confirm and summarize understanding for solving a system of equations. Do not expect students to necessarily master the process of solving systems of equations at this point—even via an informal method of testing points. The Overview is intended to serve as an introduction to the basic ideas of systems of linear equations. Many opportunities for working with systems problems are built into upcoming lessons. Students should check and update as necessary their answers on page 8 of the student booklet during this part of the lesson.
Topic 7: Formulating and Solving Systems

1.3 Process Homework

With your partner, process the homework that was due today. Use the Homework Processing Routine for the program. A copy of the routine is included below.

Homework Processing Routine

With Your Partner
Discuss/review the homework:

- Compare your answers.
  - Did you both get the same answers?
  - If not, think things through—which solution is correct?
- Compare how you solved the problem.
  - Did you both use the same approach?
  - If you used different approaches, look at the other person’s approach. How is it similar to yours? How is it different?
- How would we explain this problem if we were asked to do so?

By Yourself

- Mark a “stoplight” indicator at the top of the homework assignment:
  - Green = I understand all of the ideas in the homework.
  - Yellow = I understand some/most of the ideas.
  - Red = I don’t understand most of the ideas in the homework.
- Correct problems that you got wrong.
  - Make sure you understand why you got it wrong.
  - Don’t write down a correction if you don’t understand why the answer is correct.
  - USE A RED PEN TO MAKE YOUR CORRECTIONS.

Finish

- Place your corrected homework assignment in your two-pocket folder. Be sure to place it on the IN side.
- Remove any assignment on the OUT side of the folder and place it in your notebook. Your teacher has looked at these assignments to monitor your understanding and has checked them in.
- Be ready to participate in a brief whole-class discussion of the homework assignment and/or to place a solution on a white board if you are asked to do so.
ACTIVITY 3: Process Homework (10 minutes, p. 9)

1. Students should use the Homework Processing Routine (see p. 9 in the student booklet) to process the homework that was due today as well as to process the homework each day.

2. You may want to review the routine with students. [You can project student p. 9 using the electronic pdf file for Day 1.]

3. A suggestion is to model each step to the entire class with one pair of students, being very explicit about what each student is supposed to do and how to do it.

4. Circulate around the room as students engage in discussion with their partners and as they make corrections to their assignments. Use the data that you take in to determine if you want to do a quick follow-up to the partner conversations. For example, you might ask one or two students to explain or present one of the homework problems.

5. Try to keep the homework processing brief and to the point. Collecting and reviewing the homework, including looking at the stoplight colors that students have placed on the assignment, should help inform you as to whether more detailed follow-up or review might be needed. A follow-up discussion can be done now or can be planned for an upcoming class.

6. You can also use the routine in connection with the Staying Sharps, whether you use them in class or assign them for homework.

7. Note that one of the purposes of the homework processing routine is to build a culture of self-regulation and responsibility among students in terms of their own learning in addition to providing you with useful feedback.
The combinations that satisfy the condition:

\[ l = 8, w = 1 \quad l = 5, w = 4 \quad l = 2, w = 7 \]
\[ l = 7, w = 2 \quad l = 4, w = 5 \quad l = 1, w = 8 \]
\[ l = 6, w = 3 \quad l = 3, w = 6 \]

Some combinations that satisfy the condition (there are others):

\[ l = 24, w = 12 \quad l = 8, w = 4 \quad l = 3, w = 1.5 \]
\[ l = 14, w = 7 \quad l = 7, w = 3.5 \quad l = 2, w = 1 \]
\[ l = 10, w = 5 \quad l = 6, w = 3 \]
ACTIVITY 4: Building a System of Equations Problem (20 minutes; pp. 10-11)

1. Overview: This activity is intended to reinforce students’ understanding of the basic idea behind a system of equations as a set of conditions and the idea that a solution to a system of equations must satisfy both (or, in the case of more complicated systems, all) conditions.

2. This activity takes students through the development, or building, of a system by posing a problem of fencing a rectangular garden. Students are first presented with a single condition that the perimeter of a rectangle must equal 18 feet. They are asked to come up with a list of possible combinations for the length and the width of the rectangle that meet this condition. Students are then presented with a new problem (i.e., they consider it separately from the condition of the perimeter of 18 feet). The new problem asks students to list possible combinations for the length and width of the rectangle, with the length double the width. Finally, students are told that both conditions need to be met. The expectation here is that students would search their lists for a combination for the length and width of the rectangle that appears on both lists. It might be necessary for students to expand their lists at this point if they do not find a combination that appears on both lists—this is an important reminder that you can give to students if they are having a hard time coming up with a combination for Part 3 of the activity. By considering the conditions separately—and then putting them together—students will gain a fundamental insight into how systems of equations work.

3. To launch this activity, read through Part I as a class. [You may want to project student pages 9-10 using the electronic pdf file for Day 1.] At this point, you may have a very brief whole-class discussion to ensure that students understand the problem. For example, you may ask if a student with his/her partner can state the problem/situation in his/her own words. Then allow students to work through the problem from beginning to end; it should be accessible to them. Students should work with their partners on this activity.

4. While students are working on the activity, you can circulate around the room, checking for understanding and asking clarifying questions. If you notice that students as a whole are struggling with parts of the activity, you can bring the class together for some quick discussion. You can ask questions such as, “Can someone tell us a possible combination that works for the situation in Part 1? In Part 2? However, allow the cognitive demand of the task to remain high (that is, do not do too much scaffolding or thinking for the students); try to ensure that solving the problem is still up to the students.

5. Process the activity with whole-class discussion. Summarize the idea behind the activity (see #1 above); a system as being a set of conditions; a solution to a system is a set of values that satisfies all (both) conditions. In wrapping up the activity, ask students if considering the conditions/equations separately—and then putting them together—helped them in their understanding of the meaning of a system of equations.
Topic 7: Formulating and Solving Systems

Part 3. In the end, it turns out that the Botany Club wants to meet each of the conditions in the above problems; that is:

- Exactly 18 feet of fencing will be used for the rectangular garden; and
- The length of the garden has to be double the width of the garden.

Can you find a solution that meets both of these conditions? Your work from Parts 1 and 2 can assist you in finding an answer. Write your answer below and state how you know your answer is correct.

\[ l = 6, w = 3 \]

Part 4. Part 1 involves one condition/one linear equation. Part 2, considered on its own, also represents one condition/one linear equation. A system of equations is created when we put the two conditions together and specify that both conditions have to be met. Often we use a brace symbol to show that the two equations should be considered together as a set or as a system of equations. Write the system of equations that defines the garden problem in Part 3 in the space below.

\[ \begin{align*}
2l + 2w &= 18 \\
l &= 2w
\end{align*} \]
Topic 7: Formulating and Solving Systems

1.5 Square Box Problems as Systems of Equations

You can think of solving a system of equations as trying to solve a puzzle or riddle. In fact, you have been solving some problems since the opening days of this course—Square Box Problems—that can be thought of as systems of equations problems. Although you likely did not use a system of equations approach to solve the problems, the type of reasoning used matches the reasoning used to solve systems problems. In this activity, you will work to connect the Square Box Problems to systems of equations problems; in making this connection, the goal is to develop a deeper understanding of systems of equations.

The Square Box to the right is labeled to remind you how these problems work.

Fill in the missing entries in the Square Box Problem below:

```
product  #  #  sum
32       ?  ?  18
```

Now, suppose we had used variables instead of question marks in the diagram (see below). We can write a system of equations to model the situation:

```
\begin{align*}
x \cdot y &= 32 \\
x + y &= 18
\end{align*}
```

In words, the system of equations is stating:

"The product of two numbers is 32." AND "The sum of those two numbers is 18."

The job, then, is to find the numbers that meet both conditions, which you have been doing since early in the course.

Solution: \begin{align*} x &= 16 \quad \text{OR} \quad x &= 2 \\
y &= 2 \quad \text{OR} \quad y &= 16 \end{align*}
ACTIVITY 5: Square Box Problems as Systems of Equations (15 minutes; pp. 12-14)

1. Overview: The purpose of this activity is to make a connection between systems of equations and square box problems. This should provide a familiar context for students to think about and to deepen their understanding of a system of equations. Aside from providing a familiar context to think about systems of equations, another purpose of the activity is to practice the type of thinking that is required to solve systems of equations.

2. Launch the activity by asking students how the square box problems can be thought of as a system of equations. Read through p. 12 of the student booklet as a class. [You might choose to project student p. 12 using the electronic pdf file for Day 1.] Then students should work through the activity with their partners.

3. As students work on the activity, circulate around the room, checking for understanding and asking clarifying questions.

4. Briefly process the activity as a whole class. Consider assigning problems to different pairs of students and having them make brief presentations using their whiteboards. [If time is an issue, a useful strategy is to assign a problem to different pairs of students who complete the assignment early; students can arrange the whiteboards (in order) on the board ledge; students can then check their work.]

5. Some additional notes on this activity:

- Although the square-box problems do not represent a system of linear equations since one of the equations/conditions will be of the form $x \cdot y = #$, the context is still a good one to reinforce the idea of a system. You do not need to make this distinction for students at this time. A goal of the activity is to connect the thinking behind systems of equations—which students have been doing since the first days of the course with these problems—to a familiar context.

- In square box problems like the one below, the values for $x$ and $y$ can be interchanged. [For example, the answer to the problem below can be expressed as either $x = 12, y = 4$ or as $x = 4, y = 12$. This results from the fact that both multiplication and addition are commutative.] Try not to allow this to be a distraction to solving systems of linear equations.

$$\begin{cases} x \cdot y = 48 \\ x + y = 16 \end{cases}$$
Topic 7: Formulating and Solving Systems

The problems below ask you to make connections between Square Box Problems—or extensions of these problems—and systems of equations. Complete each of the problems below.

1. (a) Complete the system of equations for the Square Box Problem shown to the left:
   \[
   \begin{align*}
   x \cdot y &= 21 \\
   x + y &= 20
   \end{align*}
   \]
   (b) Solve the system of equations:
   \[
   \begin{align*}
   x &= \quad 20 \ (\text{or } 1) \\
   y &= \quad 1 \ (\text{or } 20)
   \end{align*}
   \]

2. (a) Write a system of equations for the Square Box Problem shown to the left:
   \[
   \begin{align*}
   x \cdot y &= 48 \\
   x + y &= 16 \quad \text{(or } 4) \quad \text{and } 4 \quad \text{(or } 12) \end{align*}
   \]
   (b) Solve the system of equations:
   \[
   \begin{align*}
   x &= \quad 12 \ (\text{or } 4) \\
   y &= \quad 4 \ (\text{or } 12)
   \end{align*}
   \]

3. (a) Write a system of equations for the Square Box Problem shown to the left:
   \[
   \begin{align*}
   4a &= 24 \\
   4 + a &= b
   \end{align*}
   \]
   (b) Solve the system of equations:
   \[
   \begin{align*}
   a &= \quad 6 \\
   b &= \quad 10
   \end{align*}
   \]

4. (a) Write a system of equations for the Square Box Problem shown to the left:
   \[
   \begin{align*}
   m \cdot (-2) &= -10 \\
   m + (-2) &= n
   \end{align*}
   \]
   (b) Solve the system of equations:
   \[
   \begin{align*}
   m &= \quad 5 \\
   n &= \quad 3
   \end{align*}
   \]

Continued
5. The product of two numbers is 27 and the sum of those two numbers is 12. 
(a) Write a system of equations for the stated problem. 
   \[ \begin{align*} 
   x \cdot y &= 27 \\
   x + y &= 12 
   \end{align*} \]
(b) Solve the system of equations: 
   \[ \begin{align*} 
   x &= \quad 9 \quad \text{or} \quad 3 \\
   y &= \quad 3 \quad \text{or} \quad 9 
   \end{align*} \]

6. The product of two numbers is 100 and the difference of those two numbers is 15.
(a) Write a system of equations for the stated problem. 
   \[ \begin{align*} 
   x \cdot y &= 100 \\
   x - y &= 15 
   \end{align*} \]
(b) Solve the system of equations: 
   \[ \begin{align*} 
   x &= \quad 20 \\
   y &= \quad 5 
   \end{align*} \]

1.6 Introducing Homework

Tonight’s homework assignment:
- Homework 1
- Staying Sharp 1

Notes or additional instructions based on whole-class discussion of homework assignment:
ACTIVITY 6: Introduce Homework (5 minutes; p. 14)

1. Since this is a start of a new unit, you may want to review the routine for introducing homework, which appears below.

   - Open your book to the homework assignment as directed by the teacher.
   - Record the homework assignment in your planner or notebook.
   - Read the homework as directed by the teacher (e.g., silently, partner reading, individual students reading aloud certain parts of the assignment).
   - Participate in a brief class discussion to ensure that you understand the homework assignment.
     - The teacher might ask you to state the homework assignment in your own words.
     - The teacher might ask you to think through and share with the class how you might approach a particular problem (without giving away the answer).

   NOTE THAT SPACE IS PROVIDED IN THE STUDENT BOOKLET FOR STUDENTS TO RECORD SOME REMINDERS / IDEAS IN CONNECTION WITH THE INTRODUCTION OF THE HOMEWORK.

2. Homework 1: “Chickens and Cows and Systems of Equations” (pp. 17-18 in the student booklet) has two parts. In the first part, students define variables and then use the variables to set up a system of equations based on the information/conditions in the problem. Students are not required to solve the systems. You might ask students what they could refer to if they needed some assistance in defining variables and setting up equations. [A logical reference would be the Supervisor and Assistant problem from today’s lesson.] In Part 2, students are asked to set up and solve (intuitively) some systems problems in connection with square box problems.

3. Staying Sharp 1 appears on p. 15 in the student booklet. For your reference: Several of the problems involve considering rectangles of different dimensions that yield the same perimeter, since this will continue to be a context for some systems problems throughout the unit.
1. Draw a rectangle (and label its dimensions) that meets the following conditions:
   - The perimeter is 36 units.
   - The length of the rectangle is 5 times greater than the width.

   Answer:
   \[
   \begin{array}{c}
   \text{length} \\
   \text{width}
   \end{array}
   \]
   \[
   \begin{array}{c}
   3 \\
   15
   \end{array}
   \]

2. Draw all three possible rectangles whose whole-number dimensions result in a rectangle with a perimeter of 14 units. Then circle the rectangle whose length is 2.5 times greater than its width.

   Answer:
   \[
   \begin{array}{c}
   l = 6, w = 1 \\
   l = 5, w = 2 \\
   l = 4, w = 3
   \end{array}
   \]

3. Janet sees the problem below on a quiz. What is the answer to the problem? 
   \((-6)(-5)(-2)(2) = ?\)

   Answer:
   \[
   -120
   \]

4. Thomas ran 200 yards in 40 seconds and Will ran 800 yards in 140 seconds. Who ran at a faster rate? Justify your answer.

   Answer with supporting work:
   - Thomas' rate: \(\frac{200}{40} = 5\) yds/sec
   - Will's rate: \(\frac{800}{140} = 5.7\) yds/sec
   Will ran faster because he has a faster rate/speed.

5. If \(2(4x + 3) - 12 = 3x - 6\), then \(x = ?\)

   Answer with supporting work:
   \[
   \begin{align*}
   2(4x + 3) - 12 &= 3x - 6 \\
   8x + 6 - 12 &= 3x - 6 \\
   5x &= 0 \\
   x &= 0
   \end{align*}
   \]

6. Express the following equation in slope-intercept form: \(4x - 2y = 6\).

   Answer with supporting work:
   \[
   \begin{align*}
   4x - 2y &= 6 \\
   -2y &= 4x - 6 \\
   y &= 2x - 3
   \end{align*}
   \]
Let \( k = \text{number of chickens} \) and \( w = \text{number of cows} \)

Then,
\[
\begin{align*}
3k + 4w &= 110 \\
2k + 4w &= 110
\end{align*}
\]

Let \( r = \text{number of 4-point questions} \) and \( v = \text{number of five-point questions} \)

Then,
\[
\begin{align*}
4r + 5v &= 100 \\
r + v &= 24
\end{align*}
\]

Let \( q = \text{number of quarters} \) and \( n = \text{number of nickels} \)

Then,
\[
\begin{align*}
q + n &= 14 \\
.25q + .05n &= 2.50
\end{align*}
\]

Let \( c = \text{number of ice cream cones sold} \) and \( s = \text{number of sundaes sold} \)

Then,
\[
\begin{align*}
c + s &= 114 \\
2c + 3.5s &= 301.5
\end{align*}
\]
Unit 6: Systems of Linear Equations

Homework 1

Part II. For the problems below, set up and solve a system of equations based on the Square Box Problem pattern.

1. (a) Complete the system of equations for the Square Box Problem shown to the left:
   \[
   \begin{align*}
   x \cdot y &= 36 \\
   x + y &= 20 \\
   \end{align*}
   \]

   (b) Solve the system of equations:
   \[
   \begin{align*}
   x &= 18 \text{ (or 2)} \\
   y &= 2 \text{ (or 18)} \\
   \end{align*}
   \]

2. (a) Write a system of equations for the Square Box Problem shown to the left:
   \[
   \begin{align*}
   x \cdot y &= -6 \\
   x + y &= 1 \\
   \end{align*}
   \]

   (b) Solve the system of equations:
   \[
   \begin{align*}
   x &= 3 \text{ (or -2)} \\
   y &= -2 \text{ (or 3)} \\
   \end{align*}
   \]

3. (a) Write a system of equations for the Square Box Problem shown to the left:
   \[
   \begin{align*}
   10 \cdot m &= 40 \\
   10 + m &= n \\
   \end{align*}
   \]

   (b) Solve the system of equations:
   \[
   \begin{align*}
   m &= 4 \\
   n &= 14 \\
   \end{align*}
   \]

4. (a) Write a system of equations for the Square Box Problem shown to the left:
   \[
   \begin{align*}
   a \cdot (-5) &= -20 \\
   a + (-5) &= b \\
   \end{align*}
   \]

   (b) Solve the system of equations:
   \[
   \begin{align*}
   a &= 4 \\
   b &= -1 \\
   \end{align*}
   \]